

ABSTRACTION (Gr. ἀφαίρεσις [aph'íresis]; Lat. *abstractio* — detachment, division, retention) — in ordinary language abstraction designates the attitude of someone who is detached from everyday life and does not take into account what is real. In philosophy the term abstraction designates a specific operation of the intellect consisting in detaching and retaining some property from a thing. This property serves as the basis upon which the intellect forms a cognitive image or concept (an abstraction) of the thing. The term abstraction also designates the method for constructing the object of intellectual cognition in general, while in particular it concerns the object proper to particular sciences (natural, mathematical and philosophical sciences). Boethius was responsible for the translation of the Greek word ἀφαίρεσις into Latin as *abstractio*.

ABSTRACTION AS AN OPERATION OF THE INTELLECT. The theory of abstraction was formulated by Aristotle in the context of a discussion with Plato on the ontological status of numbers. The theory of abstraction underlies Aristotle's conception of realistic cognition. He saw the need to reconstruct the vision of the world as a whole in a different way than his predecessors, especially Plato. In this reconstruction, he had to reject the Platonic conception of knowledge by anamnesis (memory) and to formulate the conception of abstract cognition (cognition that detaches and formulates). In his conception of the intellect, Aristotle parted ways from the Greek naturalists who had reduced the intellect to a primordial particle that had a certain shape, and from Plato who identified the intellect with the spirit-soul. Aristotle conceived the intellect (or the reason) as a faculty of the soul ("by mind I mean that part by which the soul thinks and forms judgments", *De an.*, 429 a 20-25). The intellect "can have no other characteristic except to be in potency [to knowledge]" (*ibid.*, 429 a 20-25). His discovery of the intellect as a faculty of the soul was one of the most important accomplishments of Aristotelian anthropology and epistemology. The soul remains as in Plato the "place of forms" with this difference, that "this does not apply to the soul as a whole, but only in its thinking capacity, and forms occupy it not actually but only potentially" (*ibid.*, 429 a 25-30). The intellect as the soul's faculty of cognition appears during the process of cognition in its function of retaining, detaching (abstracting) and dematerializing forms in order to make them objects of cognition. These forms characterize the necessary or non-necessary arrangements of contents in that which is individual and concrete (*ibid.*, 429 b). In Aristotle's conception, the senses and intellect are included in this process of abstract cognition, while Aristotle's predecessors conceived of them as opposed to each other.

Aristotle developed his theory of abstract cognition and the conception of abstraction in the context of his discussion with Plato on the nature and genesis of mathematical objects (*ibid.*, 429 b; *Met.*, 1086 a). According to Aristotle, mathematical objects (numbers and shapes) are created by the intellect by detaching (abstracting) and retaining the form that characterizes the relation of quantitative order in what is individual and material. In this way mathematical-geometrical objects are created (numbers, shapes, geometrical solids), although these do not exist on their own (as Plato held). Aristotle pointed to this operation of the intellect, which was characteristic of the soul's rational faculty, and by which "objects [of knowledge] are separable from their matter" (*De an.*, 429 b), as one of the first and most basic operation of the intellect in general. By this operation we create the objects of scientific cognition, which objects are general forms abstracted from individual and concrete things, and we form the cognitive images of things (*ibid.*, 429 b).

To explain the process whereby concepts arise and the objects of intellectual cognition are formed, Aristotle distinguished the so-called passive intellect and the active intellect within the structure of the soul's rational faculty. His distinction was in view of the functions the faculty performs and the way it is joined with the soul (ibid., 430 a). The passive intellect is the "place of forms" and it is part of the soul's internal structure, and the soul is the first form of an organic body. The active intellect is not joined with the body, since it is "separable, impassive and unmixed" (ibid., 430 a15-20). It is a "power" (a light) imparted to the passive reason, and by this light the reason grasps (or conceives) the intellectual image of the thing.

After Aristotle, philosophers distinguished between abstraction as the operation whereby a form is detached (or separated) from the individual and concrete thing and retained in the intellectual as an intellectual cognitive form, and abstraction conceived as the operation whereby concepts are joined and divided (this operation is called judgment) or the operation on the extensions of concepts (universalization).

During every operation of abstraction the known thing's act of existence is passed over. The act of existence is always individual. We retain the abstracted general form which characterizes the arrangement of contents. The operation of abstraction aims at producing cognitive forms of things that reflect an essential or accidental arrangement of contents.

**KINDS OF ABSTRACTION.** Various kinds of abstraction can be distinguished depending upon the criteria adopted. If we consider the intellect's function of detaching the universal from the matter of the thing known, we may distinguish universal and formal abstraction. We are dealing with universal abstraction (*universalis*), which is sometimes called total abstraction (*totalis* or *totius*), when the intellect passes over individual features in the things it knows and retains that which is universal. (*S. th.*, I, q. 40, a. 3 resp.). In this way the intellect is able to produce the so-called universals, such as animal, man, tree etc.. We are dealing with formal abstraction when the intellect detaches form from matter, whether from individual matter or general matter, or generally from matter as such. When the form is detached from individual or general matter, "both [the concepts of matter and form] remain in the intellect" (ibid, I, q. 40, a. 3, resp.). When a form is generally detached from matter, only the form retains its concept in thought.

In the case where the form is detached from the matter of the individual thing, we are dealing with so-called physical (or natural) abstraction. For example, we retain in our mind the concept of shape (form) and the concept of wood (matter). When in turn the form is separated from general matter, we are dealing with mathematical abstraction. For example, we retain in our mind the concept of the number one (form) and the concept of matter as its substratum. When form is detached from matter as such and conceived as an element of the nature of being in a particular being (assuming here an Aristotelian understanding of being), we are dealing with metaphysical abstraction. In this case we retain in our mind only the concept of form as that "whereby something is what it is".

If we take as our criterium for division the degree of reflection in the operation of abstraction, we may distinguish between pre-scientific and scientific abstraction. Pre-scientific abstraction, also called spontaneous abstraction, describes an operation of detachment and division where the selection of the aspect apprehended is not reflective to any great degree. Scientific reflection describes an operation of detachment and division that is precisely reflective and determined with regard to its aim and method.

The division of abstraction into subjective and objective is made from the point of view of the knowing subject and the object of knowledge. The basic operation of the intellect which has the aim of creating concepts is described as subjective abstraction. The operation of the intellect directed at singling out the proper objects of intellectual cognition is described as objective abstraction.

In scholastic logic we encounter the division between negative and positive abstraction. This division is based on the way judgments are formed. In positive abstraction we may create judgments by the composition or joining of concepts, and in negative abstraction we create judgments by dividing or disconnecting concepts. Then we think that something is in something else, or that it is separate from the other (*S. th.*, I, q. 85, a. 1, ad 1). In contemporary logic we may encounter the division between generalizing and specifying abstraction. The criterium for this division is the operation on the extensions of names. We may also encounter the division between quantifying and formalizing abstraction, which is based on how we pass from the name of an individual (e.g., an animal) to the name of a property (e.g., animality), or from a sentence or name to a formula of a sentence or name. By distinguishing the various criteria of division, we may formulate the various typologies of abstraction that we actually encounter in philosophy.

**ABSTRACTION AS THE METHOD WHEREBY THE OBJECT OF INTELLECTUAL KNOWLEDGE IS FORMED.** In the Aristotelian tradition of philosophy, it is common to distinguish two types of abstraction conceived as the method whereby the object of intellectual knowledge is formed: pre-scientific and scientific abstraction. "For the stone does not exist in the soul, but only the form of the soul", writes Aristotle (*De an.*, 431 b).

Pre-scientific abstraction is one of the first and most basic ways of forming the object of intellectual knowledge. As an operation of concept formation it is weakly reflected with regard to method, for it is not performed according to a previously established aim and a definite procedure (detachment). The object of intellectual knowledge that is produced is vaguely grasped. This happens because at this stage in the formation of the object of knowledge, abstraction is aimed at grasping only the features by which we may distinguish one thing from another, and this is usually for practical purposes. Furthermore, in pre-scientific abstraction the major tendency is to form simple mental schemas of things rather than to extract the contents of things. These schemas determine the extension of the object of intellectual knowledge that is produced depending on whether the schematic apprehension of a thing is performed upon the background of some feature (e.g., a bipedal being, etc.), or in the relation of one thing to others (e.g., people, animals, trees), or upon the background of a feature that is common to many things (things that move, things that are motionless etc.).

Scientific abstraction is a methodically organized and reflected procedure that is directed at forming the proper object for a given science. For this reason it is also called objective abstraction. This kind of abstraction is performed according to a definite method and a definite goal. The image of the thing known as a whole is broken down into aspective images created according to the needs of scientific knowledge (*Met.*, 1061 a 28 - b 10).

The possibility of distinguishing the proper object for a given science by scientific abstraction underlies the distinction among the three types of scientific abstraction that are proper to the distinction of three domains of science.

*Physical abstraction.* In physical abstraction the form detached from the thing's individual matter remains when the object of knowledge is constructed. This form reflects the image of the thing's qualitative endowment, e.g., weight, lightness, hardness, warmth, cold etc.. The object formed expresses the essence of that which exists individually and materially, and so it may express that which is corporeal, vegetative, sensitive or rational. This kind of abstraction is the foundation for distinguishing the entire family of natural sciences. According to Aristotle, one of the tasks of these sciences is to know individual things under the aspect of change, and thus also under the aspect of vegetative, sensitive and rational life, but not under the aspect of their existence (*Met.*, 1061 b 6-7).

*Mathematical abstraction.* Aristotle primarily associated the distinction of the object of mathematics with this kind of abstraction, since, as he stated, "the mathematician makes a study of abstractions" (*Met.*, 1061 a 29-30). By abstraction, mathematical objects are divided from the matter that falls under the senses. These objects retain in themselves "only quantity and continuity in one, two or three dimensions" (*Met.*, 1061 a 33-34). The mathematical images of things such as numbers, lines, shapes or geometrical solids manifest the essence of quantity and continuity as quantitative relations that are the fundamental determination of material things. Mathematical abstraction underlies the distinction of the mathematical and geometrical sciences, for as Aristotle explains, these sciences investigate objects with respect to "quantity and continuity [...] their respective positions and properties, and in others their commensurability or incommensurability, and in others their ratios" (*Met.*, 1061 a 34 - b 2).

*Metaphysical abstraction.* The purpose of metaphysical abstraction is to form the proper object for philosophical knowledge. This object is form separated from matter as such, and the form indicates that by virtue of which a thing is what it is. Metaphysical abstraction lies at the basis of the distinctness of the philosophical sciences which have the role of "studying the properties of being as being and the contraries of being as being" (*Met.*, 1061 b 4-5). Philosophical (metaphysical) studies differ from other studies in that individual and concrete things are studied "not insofar as each of them has some definite attribute, but [...] in so far as each of them is a being" (*ibid.*, 1061 b 26-29).

Aristotle arranges in an hierarchy the methods of abstraction that are the basis for forming the proper objects of the natural, mathematical and philosophical sciences. He constructs the hierarchy with a view to the scope of the knowledge gained in each. He distinguishes metaphysical abstraction and metaphysics as the first science, because the object of philosophy that is formed as a result of metaphysical abstraction is the most universal. It includes within its scope everything that exists. For this reason Aristotle remarks that "knowledge of nature and mathematics should both be regarded merely as a part of wisdom" (*ibid.*, 1061 b 33-34), for these sciences study things either insofar as they are subject to change or under the aspect of quantity and continuity, but they do not study things insofar as they are being. Only first philosophy (metaphysics) is concerned with things "insofar as they are being and nothing else" (*ibid.*, 1061 b 33), and therefore it alone merits the status of the first science.

THE HISTORY OF THE THEORY OF ABSTRACTION. We may search for the foundations of the threefold division of Aristotelian methodic abstraction (physical, mathematical, and metaphysical abstraction) in the Platonic conception of knowledge as noetic (contemplative), dianoetic (mathematical) and doxal (probable). The difference in Aristotle's conception was that for Aristotle individual and concrete things were the basis and aim of abstract knowledge. Plato had rejected this as impossible. Furthermore, Platonic

knowledge had an intuitive character as the memory (anamnesis) of innate ideas. For Aristotle, cognition is a process in which by methodically reflective abstraction we formulate a cognitive image of things. This image, although it is detached from matter, still has a basis in individually and concretely existing things. It should be noted, however, that in Aristotle the term "abstraction" is directly associated with the formation of mathematical objects which in his opinion result from abstraction, and which for this reason do not exist as ideal objects as Plato held (*Met.*, 991 b 9 - 992 a 24; 1061 a 29 - b 33). Aristotle conceived of abstraction as an operation of the intellect whereby the intellect retains separated and distinct aspects from individually and concretely existing things and omits other aspects. We encounter this sort of abstraction not only in mathematical knowledge, but also in every kind of scientific knowledge.

The Aristotelian theory of abstraction was variously interpreted and assigned various functions over the history of philosophy. Avicenna (10th and 11th century), for example, interpreted Aristotelian abstraction in a Platonic spirit. He identified the human soul with the human intellect and described abstraction as a process of illumination that the divine Intelligence imparts to the soul. As a result of this illumination individual things are stripped of their matter and accidents. The result of abstraction (or illumination) is that the essence or nature is stripped of everything that is individual and the essence is imprinted upon the rational soul. Here there is no transition from the imagination to the intellectual image or concept. The reason receives the intellectually knowable form from the divine Giver of forms (*Dator formarum*). Abstraction is the process by which the soul receives the corresponding cognitive form of the thing. The process of abstraction and the acquired cognitive forms of things are not the result of autonomous acts of the soul's rational faculty, but they come from the separate Intelligence that acts upon the soul. Cognition and learning are the acquisition of the ability to join oneself with the Intelligence from which the soul receives an understanding of the thing along with the cognitive form.

Thomas Aquinas developed the Aristotelian theory of abstraction and indicated two kinds of abstraction. In one kind of abstraction the reason grasps or joins features that are separate from each other (*S. th.*, q. 85, a. 1 ad 1). In the other kind, the reason divides features that in reality constitute one thing. The first kind of abstraction occurs when we construct judgments, and the second occurs when we construct concepts.

The abstraction that occurs in the construction of concepts can happen in two ways: by detaching the universal from the particular, and by detaching the form from matter (*S. th.*, I, q. 40, a. 3 resp.). In the first case the reason rejects the individual and particular in the thing and forms universals. Also, it performs operations on the scope of concepts and creates a series of logical concepts. In the second case, abstraction consists in detaching the form from matter — whether this is individual matter (natural abstraction), general matter (mathematical abstraction) or in general matter as such (metaphysical abstraction) in the Aristotelian understanding of being.

In abstraction as it occurs in judgment, Aquinas distinguishes two separate operations: a judgment that consists in the composition and division of concepts (*ibid.*, I q. 85, a. 1, ad 1), called judgmental abstraction, and a judgment consisting in stating or affirming the existence of a thing or some property of a thing, which was later called separation (metaphysical separation). According to Thomas, neither conceptual abstraction nor judgmental abstraction is a proper method for obtaining the proper object of metaphysics, which is "being as concretely existing", for the cognitive image (concept) of "being as existing" cannot be

limited to a grasp of any aspect of content, but it must also include the aspect of existence (*De Trinitate* q. 5, a. 3).

In this way Thomas, as one of the first of Aristotle's commentators, rejected Aristotle's conception of metaphysical abstraction and searched for another method for forming the proper object of metaphysics (separation). He restricted the Aristotelian type of abstraction to application in the natural and mathematical sciences, since it was obviously inadequate for forming the proper object of a realistic metaphysics.

Among scholastic authors we encounter various ways of dividing abstraction depending on the criterium chosen by a given author. Cajetan (Cardinal Thomas de Vio) divided abstraction into formal abstraction of the first degree, also called partial abstraction, which is interchangeable with Thomas' total abstraction. The next type was formal abstraction of the second degree, which Cajetan called total abstraction, and this corresponds to Thomas' formal abstraction. The same abstraction is called total or partial because the essence of a thing may be expressed by the form of the whole (*forma totius*, e.g., humanity) or by the form of a composite being where the form is conceived as the form of a part (*forma partis*, e.g. the rational soul). Hence certain authors began to distinguish between *abstractio partialis* and *abstractio totalis*, although we are basically dealing with the same type of abstraction, namely with the type of formal abstraction that Thomas called *abstractio totius*.

With W. Ockham there was a new stage in the interpretation of Aristotelian abstraction. He accepted the scholastic division of intuitive and abstract cognition, but with this difference: while the scholastics made this distinction from the point of view of the object (the object of intuitive cognition was the individual thing, and the object of abstract cognition was the form detached from matter), for Ockham this division concerned the nature of the knowing subject. When a sensory intuition arises, the intellect is made capable of abstract cognition. Ockham rejected the existence of a cognitive form as a *medium* for the cognition of the individual and concrete thing. The reason does not produce a general cognitive form; generality is something that arises in the intellect as a result of the natural action of individual things which the intellect does not necessarily create. Individual things are similar, and so they produce common images. The common image, however, is not a general form, but a lack of distinctness. It is *natura* that creates general concepts in the intellect, but it creates them in a mysterious way ("*Dico quod natura occulte operatur in universalibus*", *Liber sex principiorum*, I, 4; ed. D. Van Eynde, 10). We know only the results of this operation.

The next stage in the reformulation of the Aristotelian conception of abstraction was the philosophy of F. Suarez, the representative of what is called the second scholasticism. His position was similar to Ockham's. He stated that everything that exists must be something individual, and the reason initially apprehends only individual beings, and it apprehends generalities afterward. In the place of the Aristotelian distinction between the formal concept and the objective concept (leaving the concrete thing to the side), Suarez introduced a distinction between the existing thing and its cognitive image. The formal concept is the act whereby the intellect apprehends a thing or an idea. This act is the concept, since the intellect creates it; it is a formal concept, since as a form of the mind it represents a known object. The objective concept, on the other hand, is the thing or idea that the formal concept directly apprehends. The reason possesses one formal concept of a thing which is in reality and in thought different from other formal concepts of things. At the moment when the intellect encounters the word "being" (or "thing", "animal", "man"), its attention is not

distracted toward other concepts, but is held to some one thing, e.g.: to the nature of being in a being (or animality, humanity etc.). There is a strict connection between formal and objective concepts. This is expressed in the fact that only one objective concept corresponds to one formal concept ("*uni conceptui formali unus conceptus obiectivus necessario respondet*", *Disputationes metaphysicae*, II, 2, 3). According to Suarez there are two orders in the objective concept: the intentional order (of thought), and the ontic order (of reality). The cognition of being thus does not consist in detaching the form from the matter of the individual thing or in creating universals on the basis of the individual, but it consists in apprehending the objective concept (namely the thing) in differentiation (in multiplicity — *abstractio adaequate non praecisiva*) or in non-differentiation (in unity - *abstractio adaequate praecisiva*). This may remind us of the process of passing from the name ("man", "being") to its quality ("humanity", "the nature of being"). Abstraction is nothing other than the movement of the reason from what is differentiated and many to what is simple and one (ibid., II, 2, 14).

Descartes rejected the Suarezian distinction between the objective and the formal concept and focused upon the subjective concept, which was the clear and distinct idea. The idea is not a product of abstraction. Knowledge itself is reduced to an operation of analysis performed on ideas. In this way the philosophy of the subject began, and the philosophy of the object was rejected.

In modern and contemporary philosophy the Aristotelian understanding of abstraction has been for the most part rejected, whether abstraction as a spontaneous operation or the methodical abstraction whereby the intellect forms a proper object of knowledge. The term "abstraction" has been replaced by the terms "construction", "projection", "generalization", "association", and "analysis". The operation of abstraction has been conceived as the "creation" of meanings that the intellect creates by virtue of its own nature.

P.H.E. Gohlke, *Die Lehre von Der Abstraktion bei Plato und Aristoteles*, Hi 1914; A. Dondeyne, *L'abstraction*, RNSP (1938), 5-20; G. Cala Ulloa, *Il problema dell'astrazione*, Pd 1947; M.D. Philippe, *Abstraction, Addition, Séparation chez Aristote*, Rthom 48 (1948), 461-479; G. Von Riet, *Le théorie thomiste de l'abstraction*, RPL (1952), 353-393; M.D. Philippe, *Abstraction, Addition, Séparation dans la philosophie d'Aristote*, RThom 45 (1945), 461; G. Siewerth, *Die Abstraktion und das Sein nach der Lehre des Thomas von Aquin*, Sa 1958; J. Owens, *The Doctrine of Being in the Aristotelian "Metaphysics"*, Tor 1951, 1957<sup>2</sup>, 238-241; E.D. Simmons, *The Thomistic Doctrine of the Three Degrees of Formal Abstraction*, Thom 22 (1959); L.B. Geiger, *Abstraction et séparation d'après st. Thomas*, in: *Philosophie et spiritualité*, P 1963, I 87-124; L. Vincente, *De modis abstractionis iuxta St. Thomam*, DThP 66 (1963), 34-64; K. Von Fritz, *Die ἀφαιρέσις bei Aristoteles*, Mn 1964; J. Weinberg, *Abstraction, Relation and Induction*, Wisconsin 1965; T. Czeżowski, *Filozofia na rozdrożu* [Philosophy at the crossroads], Wwa 1965, 106-11; Krapiec Dzieła [Works] VII; E. Mikkola, *Die Abstraktions Begriff und Struktur*, He 1966; A. Maryniarczyk, *Spór o metodę poznania realistycznego. Abstrakcja czy separacja?* [Dispute on the method of realistic cognition. Abstraction or separation?] in: *Zadania współczesnej metafizyki. Poznanie bytu czy ustalenie sensów?* [Tasks of contemporary metaphysics. The cognition of being or the establishment of meanings?] Lb 1998.

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ABSTRACTION OF CLASSES — we call the set of all and only those elements of set  $A$  which possess the relation  $R$  to the element  $x$  the class of the abstraction of the equivalence relation  $R$  in the set  $A$  where the class is indicated by the element  $x$ .

We write the above definition symbolically in the form:

$$y \in [x]_{R,A} \equiv y \in A \wedge yRx$$

or

$$[x]_{R,A} = \{y \in A : yRx\}.$$

An example: the class of the abstraction of the relation of parallel direction of straight lines indicated by a given straight line is the set of all and only those straight lines which are parallel to that line.

Two different classes of abstraction of an arbitrary relation of equivalence in a given set are exclusive, and the sum of all the classes of abstraction is always equal to the set. Therefore, one may formulate a proposition of the so-called principle of abstraction: the set of the classes of abstraction of the equivalence relation  $R$  in set  $A$  is a logical division of set  $A$ .

Certain properties of objects are defined as classes of abstraction of an equivalence relation  $R$  (definitions by abstraction). Example: the weights of bodies are the classes of abstraction of the relation of equality of weight in the set of all bodies (in the set of bodies, the class of bodies of the weight of 1 kg and the class of bodies of the weight of 2 kg are examples of classes of abstraction); the weight of body  $x$  is the class of abstraction of this relation indicated by body  $x$ , and so it is the class (the common property) of all bodies equal in weight with body  $x$ .

THE OPERATOR OF ABSTRACTION — the symbol of operation " $\{ : \}$ ", which together with a joined individual variable and one propositional expression creates the name of a set, e.g.  $\{x : x^2 = 4\}$ , means the set of such  $x$  that  $x^2 = 4$ .

The definition of the operator of abstraction for formulas of one argument may be written as follows:

$$y \in \{x : \Phi(x)\} \equiv \Phi(y),$$

where  $\Phi(x)$  is a propositional form of the free variable  $x$ . This definition may be read as follows:  $y$  belongs to the set defined by the form  $\Phi(x)$  when and only when  $y$  completes this form.

For formulas of two arguments the definition of the operator of abstraction is the expression:

$$\langle z, u \rangle \in \{\langle x, y \rangle : R(x, y)\} \equiv R(z, u).$$

Analogous expressions may be written for arguments with a greater number of arguments.

From the linguistic point of view, the operator of abstraction is a grammatical tool that



transforms an expression from a category of predicates into an expression from the category of names (e.g., the predicate "is a philosopher" into "the set of philosophers"). This provides various possibilities for characterizing sets, comparing them among themselves and performing operations on sets, and as a result this makes more precise the concepts expressed by the predicates.

K. Ajdukiewicz, *Język i poznanie* [Logic and cognition] Wwa 1960; 1985<sup>2</sup>, I 222-242; idem, *Logika pragmatyczna* [Pragmatic logic] Wwa 1965; 1975<sup>3</sup>; L. Borkowski, *Logika formalna*, Wwa 1969, 1977<sup>2</sup>; Lfor 58-59; L. Borkowski, *Wprowadzenie do logiki i teorii mnogości* [Introduction to logic and the theory of plurality], Lb 1991.

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